MORE EXAMPLES FOR SECTION 1.6

Example 1. $y' = y + e^{2x}$.

Solution. We first transform this equation into the form $y' + p(x)y = q(x)$.

$$y' - y = e^{2x}.$$

Then $p(x) = -1$, and $q(x) = e^{2x}$. So the integrating factor is

$$I(x) = e^{\int p(x)\,dx} = e^{-x}.$$

This gives

$$(e^{-x}y)' = e^x \Longrightarrow e^{-x}y = \int e^x\,dx + C \Longrightarrow y = e^{2x} + Ce^x.$$

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Example 2. $xdy = (x^2 - y)dx$.

Solution. As in the previous example, we transform this equation into the form $y' + p(x)y = q(x)$.

$$y' + \frac{1}{x}y = x, \quad p(x) = \frac{1}{x}, \quad q(x) = x.$$

As computed in an example given today, the integration factor is

$$I(x) = e^{\int p(x)\,dx} = x.$$

Then we have

$$(xy)' = x^2 \Longrightarrow xy = \int x^2\,dx + C \Longrightarrow y = \frac{x^2}{3} + \frac{C}{x}.$$

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Example 3. $xy' + 2y = 4\cos x$

(This example was covered during the lecture for Section 144)

Solution. W transform this equation into the form $y' + p(x)y = q(x)$ and obtain

$$y' + \frac{2}{x}y = \frac{4\cos x}{x}, \quad p(x) = \frac{2}{x}, \quad q(x) = \frac{4\cos x}{x}.$$

So the integration factor is

$$I(x) = e^{\int p(x)\,dx} = e^{\int \frac{2}{x}\,dx} = e^{\ln x^2} = x^2.$$

Then we have

$$(x^2y)' = 4x\cos x \Longrightarrow x^2y = \int 4x\cos x\,dx + C \Longrightarrow y = \frac{4\sin x}{x} + \frac{4\cos x}{x^2} + \frac{C}{x^2}.$$

While evaluating the integral $\int 4x\cos x\,dx$, we have used the integration by parts:

$$\int 4x\cos x\,dx = 4x\sin x - 4\int \sin x\,dx = 4x\sin x + 4\cos x + C$$

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